

FREE-SPACE AND EXTERNAL  
HEAT-TRANSFER FLUCTUATIONS  
IN A FLUIDIZED BED

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Experimental results are used to show that fluctuations in the free space near the wall have little effect on the local heat-transfer coefficients.

If fluidization is nonuniform, the free-space and gas-speed fluctuations near the heat-transfer surface cause fluctuations in the local heat-transfer coefficients. We have examined the local heat transfer and recorded the fluctuations in the surface temperature of a transducer, the bed porosity, and the gas speed. The hydrodynamic circumstances were also recorded by cinephotography with an SKS-1M-16 camera, which worked in synchronism with an N-700 loop oscillograph. Some of the results have previously been published [1-5] along with a description of the methods of examining the structural and hydrodynamic characteristics of fluidized beds.

We estimated the effects of temperature and heat-transfer coefficient fluctuation as in [6-11] by means of a thermal anemometer whose sensitive element was a platinum foil of thickness  $5 \mu$  carrying direct current. Figure 1 shows the design and measuring circuit, and it indicates that the foil did not produce perturbations, while the rigid mica substrate eliminated errors arising from mechanical effects. The dc component of the output signal was balanced out with the SAV adjustable-voltage source and was recorded by a millivoltmeter with a long response time, while the pulsating component was passed to the oscillograph. We used various transducers with different foil lengths.

The output signal was calibrated in degrees by means of a TS-16 water thermostat; the transducer carried a current of 0.5 A, which produced scarcely any heating of the foil in relation to the water. To each set temperature between 20 and 90°C there corresponded a definite position of the spot  $\Delta h$  and a definite reading of the millivoltmeter, the latter indicating the out-of-balance voltage from the bridge. In the working state,  $I=I_w$ , and the instantaneous value of the excess temperature  $\vartheta$  is given by the following (the balancing temperature was  $t_{bc}$ ):

$$\vartheta = \bar{\vartheta} + k\Delta h \frac{0,5}{I_w} . \quad (1)$$

The value of k for foil  $20 \times 2.5$  mm was 0.59 deg/mm, while for a foil of length 10 mm it was 0.91 deg/mm.

The main experiments were performed with the transducer vertical and placed in the center of an apparatus of cross section  $30 \times 150$  mm and of height 400 mm. Checks on an apparatus of diameter 150 mm showed that the character and amplitude of the fluctuations were virtually the same. Particles of firebrick, corundum, glass, and polystyrene of diameter 0.12-0.85 mm were fluidized with air at W of 1-5; the critical velocity was taken as the value corresponding to the onset of fluidization of the entire bed. Fluidization began near the transducer at a gas speed 70-80% of the critical value.

When we processed the recordings, we used the fact that the heat transfer may be considered as quasi-stationary if the stabilization time for the thermal boundary layer is less than the heating or cooling times for the bodies [12, 13]; under these conditions, the rate of external heat transfer can be characterized by means of

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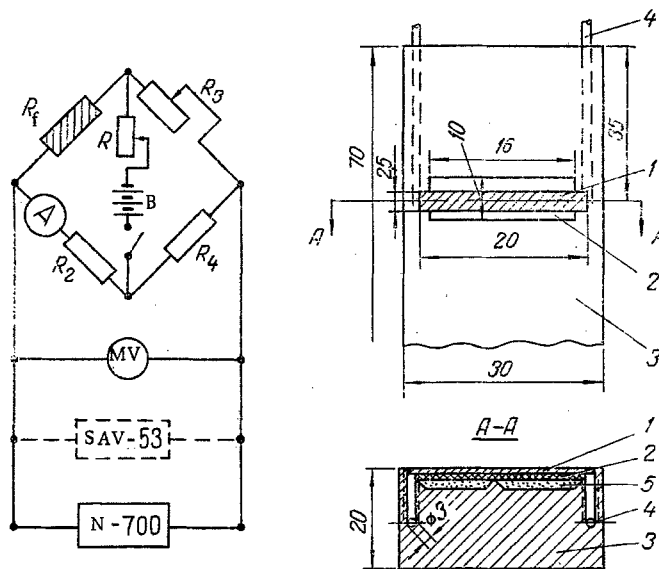


Fig. 1. Transducer circuit and design characteristics.  $R_f=0.195 \Omega$  (foil length 10 mm) or  $R_f=0.390 \Omega$  (foil length 20 mm),  $R_2=1.7 \Omega$ ,  $R_3=0-30 \Omega$ , and  $R_4=15 \Omega$ ; B) 12-V storage battery; A) class 0.5 ammeter; MV) M-82 millivoltmeter; SAV adjustable-voltage source; N-700 loop oscilloscope; 1) platinum foil; 2) mica substrates of thicknesses 0.2 and 0.5 mm; 3) textolite plate; 4) copper current leads; 5) air gap, 0.5 mm.

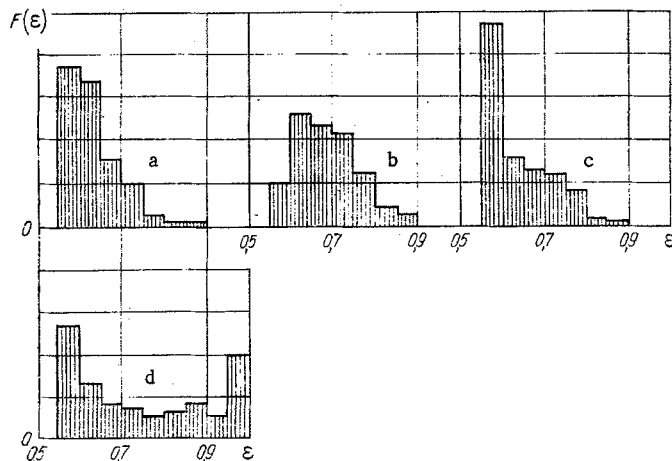


Fig. 2. Histograms for fluctuations in porosity near surface: a)  $d=0.12$  mm,  $W=1$ ; b)  $d=0.12$  mm,  $W=5$ ; c)  $d=0.50$  mm,  $W=1$ ; d)  $d=0.50$  mm,  $W=5$ .

a heat-transfer coefficient as derived from Newton's law. If  $Bi \rightarrow 0$  and the temperature gradient within the body is virtually zero, the heating and cooling of the transducer can be described accurately by the balance equation

$$cpV \frac{d\theta}{d\tau} = \alpha\theta F + Q_{\text{sou}} \quad (2)$$

Equation (2) applies if the thermophysical parameters of the foil are independent of the temperature, and if  $\alpha$  is constant, as is the temperature of the heat carrier in the given time range. We integrate (2) with limits  $\theta_{\text{min}}$  and  $\theta_{\text{max}}$  to get an expression for  $\alpha$  for a heating or cooling period:

$$\alpha_{\text{heat}} = \frac{Q_{\text{sou}} - Q_{\text{loss}}}{F \bar{\vartheta}_{\text{heat}}} - (c\rho\delta)_{\text{ef}} \ln \frac{\vartheta_{\text{max}}}{\vartheta_{\text{min}}} \cdot \frac{1}{\tau_{\text{heat}}},$$

$$\alpha_{\text{cool}} = \frac{Q_{\text{sou}} - Q_{\text{loss}}}{F \bar{\vartheta}_{\text{cool}}} + (c\rho\delta)_{\text{ef}} \ln \frac{\vartheta_{\text{max}}}{\vartheta_{\text{min}}} \cdot \frac{1}{\tau_{\text{cool}}}. \quad (3)$$

If the variations in  $\vartheta$  are small,  $\ln \frac{\vartheta_{\text{max}}}{\vartheta_{\text{min}}}$  can be replaced by  $\frac{\vartheta_{\text{max}} - \vartheta_{\text{min}}}{\bar{\vartheta}}$ ; the values of  $\tau$ ,  $\vartheta_{\text{max}}$ , and  $\vartheta_{\text{min}}$  were determined directly from the oscillograms if  $(c\rho\delta)_{\text{ef}}$  and  $Q_{\text{loss}}$  had been determined during the calibration.

The method of dynamic calibration with a pulsating air flow around the transducer (frequency 1-8 Hz) was as in [7]; flow around a plate with a given  $\bar{\alpha}$  was used to determine the heat loss through the current leads and the point of contact of the foil with the base of the transducer. The result for  $Q_{\text{loss}}$  was 22-28% of  $Q_{\text{sou}}$ .

The calculations showed that  $(c\rho\delta)_{\text{ef}}$  for  $Q_{\text{loss}} = \text{const}$  was almost independent of the amplitude of the pulsations in  $\vartheta$  (between 1 and 10°C) and of the level of the steady-state surface temperature corresponding to a given  $\alpha$  within the range from 30 to 100°C; this means that any variation in the heat-transfer coefficient from 200 to 600 W/m<sup>2</sup>·deg leaves  $(c\rho\delta)_{\text{ef}}$  constant within 10%. The exact value of  $(c\rho\delta)_{\text{ef}}$  varied with the frequency of the fluctuations from 30.7 to 26.0 J/m<sup>2</sup>·deg for the 10 × 2.5 mm foil and from 53 to 50 in the same units for the 20 × 2.5 mm foil.

Table 1 gives the results on the excess-temperature fluctuations and also values from other sources, from which it is clear that  $\alpha_{\text{heat}} < \alpha_{\text{cool}}$  and that the rate of change of temperature in the cooling period differs from that in the heating period.

However, in contrast to [7, 10, 14, 15], the difference was only small; the maximum cooling rate of 120 deg/sec was obtained on using the faster transducer in a bed containing particles of corundum of size 0.12 mm with  $W$  of 2-4; the ratio of  $\vartheta_{\text{max}} - \vartheta_{\text{min}}$  to  $\bar{\vartheta}$  did not exceed 12%, while in most of the runs it was in the range 4-6%. Results identical with ours as regards the rates of the processes have been reported previously [6, 8, 11], and in [6] and [8] the platinum foil was separated from the transducer by an air gap and had no substrate, which increased appreciably the integrating capacity of the transducer.

In [9] a thermistor was immersed in the free bed, into which gas bubbles were injected; in that case, the entire transducer could be within a gas cavity for a certain time, this then being replaced by a layer of particles only slightly fluidized, which corresponds to the conditions for maximal fluctuation in  $\alpha$  and  $\vartheta$ . However, even under such conditions the maximum deviation of  $\vartheta$  from the mean did not exceed 40%.

Table 1 also shows that only the data of [7, 10, 14, 15] differ substantially as regards the magnitude of the temperature fluctuations, although the fluidization conditions and transducer characteristics in those experiments were virtually the same as in most other studies.

Our recordings of the temperature and velocity were used with the films to establish that conditions never occurred near the surface such as to give rise to large fluctuations in  $\vartheta$ . Figure 2 shows some of the histograms for the porosity fluctuations near the wall for a layer of 5-particle-diameter width, as calculated with a Minsk-22 computer, which shows that the change in  $\varepsilon$  or in  $\vartheta$  is continuous. At low speeds ( $W < 2$ ) the most probable value of  $\varepsilon$  is 0.55-0.65.

The probability of gas bubbles free from particles was almost zero, as was the probability of close-packed particle groups.

When  $W$  was 2-4, a surface immersed in the bed was effectively covered by a dispersed medium having  $\varepsilon$  of 0.65-0.85.

High-speed cinephotography also confirmed that all possible states of dispersion occurred near the heat-transfer surface in various sequences: from a flux of gas containing a little solid to an immobile bed with gas passing through. The combination of several different states at a given instant determines the mean heat-transfer rate and also the dependence of  $\bar{\alpha}$  on the major factors. The instantaneous values of  $\alpha$  can vary only within limits corresponding to the range of variation in the instantaneous characteristics of the dispersed medium, particularly the particle concentration.

For instance, when the medium was nearly all gas, porosity 0.94-0.99, with an air speed of 0.3-0.5 m/sec, and a particle diameter of 0.140-0.165 mm, the heat-transfer coefficient was 200-250 W/m<sup>2</sup>·deg, whereas at 3-5 m/sec it was 600 W/m<sup>2</sup>·deg or more [19, 20]. In the case of fluid flows [21, 22], data from various sources ( $d$  of 0.10-0.32 mm and speeds up to 0.5 m/sec) showed that  $\bar{\alpha}$  increases from 450 to 700 W/m<sup>2</sup>·deg

TABLE 1. Measurements of Surface Temperature Fluctuations at Transducer

Source	Foil size, mm	$q_{\text{sour}}$ W/m <sup>2</sup>	$(cp\delta)_{\text{ef}}$ J/m <sup>2</sup> ·deg	$\frac{\delta_{\text{max}} - \delta_{\text{min}}}{\tau_{\text{heat}}}$	$\frac{\delta_{\text{max}} - \delta_{\text{min}}}{\tau_{\text{cool}}}$	$\frac{\delta_{\text{max}} - \delta_{\text{min}}}{\delta}$
				deg/sec	deg/sec	
Our results	2,5×20× ×0,005	36000— —36600	50—53 (meas.)	15—29	38—58	0,036—0,054
	2,5×10× ×0,005	45000	26—30,7 (meas.)	33—45	80—120	0,10—0,12
H. Mickley, D. Fairbanks, R. Hawthorn [6]	8×6,35× ×0,025	—	67,3	—	38	0,20—0,30
N. F. Filippov- skii [11]	10×4× ×0,005	13000	13,4 (calc.)	24,2	52,5	0,10
Yu. N. Dokuch- aev [8]	0,8×71× ×0,020	21000	53,8	38—57	46—78	0,18
G. Henwood [9]	20×5× ×0,0127	16800	34,1	50—260	30—320	0,15—0,40
A. P. Baskakov, V. A. Kirak- osyan, and O. K. Vitt [7, 10, 14, 15]	10×5× ×0,005	19400	20—60 (meas.)	335	780	1,90
	10×8× ×0,006	15800	16,1 (calc.)	160—210	670—880	0,58—1,84

TABLE 2. Comparison of Data on Fluctuations in  $\alpha$

Source	d, mm	w	F, mir <sup>2</sup>	$\bar{\alpha}$ , W/m <sup>2</sup> ·deg	$\frac{\alpha_{\text{min}}}{\alpha_{\text{max}}}$
[7, 14]	Corundum 0,12	1—5	50	470—495	0,07
Our results	Corundum 0,12	1—5	50	425—445	0,88

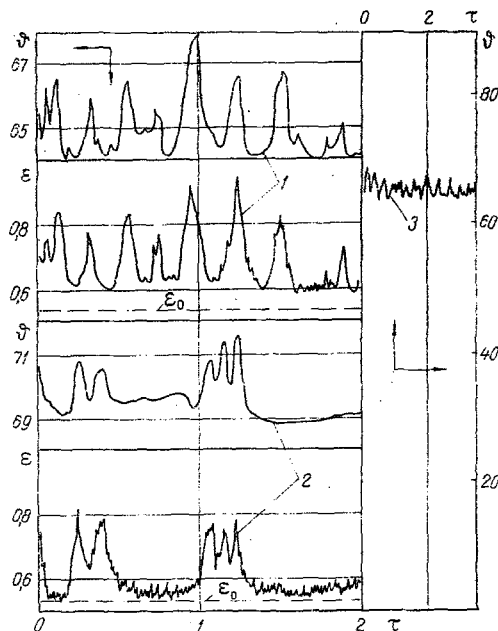


Fig. 3. Fluctuations in excess temperature  $\delta$  (deg) and bed porosity  $\varepsilon$  as functions of time  $\tau$  (sec): 1)  $d=0.32$  mm,  $W=2$ ; 2)  $d=0.50$  mm,  $W=1$ ; 3)  $d=0.32$  mm,  $W=2$ ;  $\varepsilon_0$  is the porosity of the immobile bed.

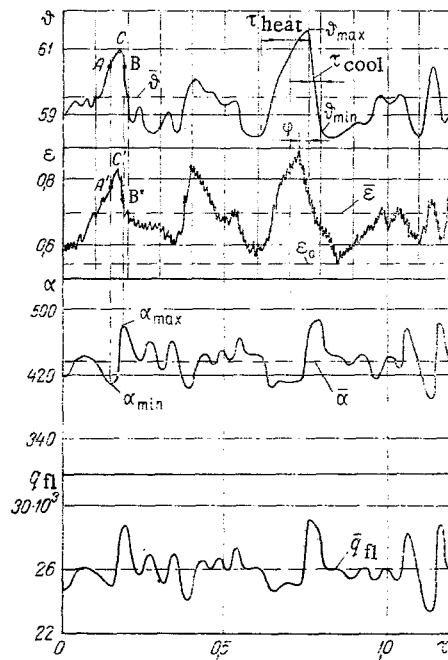


Fig. 4. Calculation of the instantaneous heat-transfer coefficient  $\alpha$  ( $\text{W}/\text{m}^2 \cdot \text{deg}$ ) and heat flux density  $q_{fi}$  ( $\text{W}/\text{m}^2$ ). Corundum particles of 0.12 mm for  $W=4$ .

as  $\bar{\epsilon}$  goes from 0.95 to 0.80, while for  $\bar{\epsilon} < 0.80$  there is even a slow fall. Further,  $\bar{\alpha}$  is 300–400  $\text{W}/\text{m}^2 \cdot \text{deg}$  for a moving bed of fine particles having  $\bar{\epsilon} = 0.7$ . These examples indicate that the instantaneous transfer coefficient differs little from  $\bar{\alpha}$  during contact with a gas bubble, i.e., one cannot assume a priori that  $\alpha_0$  is close to zero; the maximum  $\alpha$  is attained for  $\epsilon$  of 0.75–0.85 and corresponds to the contact between the surface and a vigorous flow of gas and particles, i.e., the instant of contact between the transfer surface and the hydrodynamic wake of a bubble; the heat-transfer coefficient is less than the maximum value during contact between the surface and largely immobile dense clumps.

Figure 3 shows synchronous recordings of the instantaneous  $\delta$  and  $\epsilon$ ; the two quantities fluctuate together. The cross-correlation coefficients are almost independent of the particle diameter and gas speed for  $W \geq 2$ ; with one minor exception, the correlation coefficient was greater than 0.8. On average, the output signal from the measuring system, which reflects the temperature variation in the foil, was delayed by comparison with the signal representing the porosity by 0.02 sec.

An inhomogeneous fluidized bed tends to be a self-excited oscillatory system [5], in which periodic processes are combined with random ones; the periodic oscillations in the porosity and heat-transfer rate are due to gas bubbles reaching the surface. A spectrum of random fluctuations is superimposed on these oscillations, this arising from fluctuations in the gas and particle speeds. Local fluidization foci arise initially near the transducer in a bed of fine particles as the mean gas speed increases (but provided  $W < 1$ ), and there are also oscillations and minor displacements of the individual particles, after which small bubbles begin to migrate along the surface, and the body of particles as a whole begins to move. At that instant, the heat-transfer rate increases sharply, while the oscillograph records oscillations of small amplitude in  $\epsilon$  of frequency up to 40 Hz. For  $W=1$  (Fig. 3), an air cavity appears around the lower end of the plate, with periodic breakaway of gas along the surface, which goes with the high-frequency oscillations to produce low-frequency pulsations in  $\epsilon$ . At these instants, the porosity rises to 0.75–0.80, while  $\delta$  varies within a range of 2°C. In the case of particles with  $d=0.50$  mm and  $W=1$ , there were prolonged intervals (up to 1.5 sec) during which there were no low-frequency pulsations, i.e.,  $\delta$  and  $\epsilon$  remained constant if one ignored the small-scale fluctuations due to the random motion of the individual particles (curves 2 of Fig. 3). This indicates that the close-packed particle clumps do not heat up appreciably even for  $\Delta\tau > 1$  sec, i.e., the heat transfer to the fluidized bed is mainly of convective type.

We calculated the instantaneous  $\alpha$  from (3) using oscillograms representing realizations of length 15 sec; the shortest time interval (for which one can assume that  $\alpha = \text{constant}$ ) was taken as 0.025 sec, which corresponds to the onset of the regular state under these conditions. This interval was determined by experiment from the cooling rate of the foil after a steep reduction in the electrical input.

The calculation shows that the fluctuations in  $\alpha$  relative to  $\bar{\alpha}$  did not exceed 8% throughout the range in  $d$  and  $W$  for firebrick particles of diameter 0.7 mm, or 6% corundum particles of 0.12 mm. Figure 4 shows an example of the results for  $q_{fl}$  and  $\alpha$  for small corundum particles; the curves of Fig. 4 go with the film to confirm that  $\alpha$  begins to fall slightly if a gas bubble approaches the surface element, which is accompanied by an increase in the porosity. However, there is usually particle movement between a rising bubble and the wall (i.e.,  $\varepsilon < 1$ ), and therefore  $\alpha$  for gas bubbles (part AC in Fig. 4) does not fall below  $390 \text{ W/m}^2 \cdot \text{deg}$ , i.e., is an order of magnitude higher than for steady-state flow along a shorter plate flushed by pure gas having a speed equal to the mean filtration speed. This result confirms the above conclusions on the role of the gas bubbles and casts doubt on the reliability of any model concepts in which the heat-transfer rate during the contact with bubbles is assumed to be negligible.

Figure 4 shows that the base of the bubble approaches the surface at points of the type of C, and the local gas speed rises, with the particles previously adhering to the wall or sinking down it then beginning to rise. Although the porosity at this instant is virtually as before,  $\alpha$  increases, while the heating of the foil ceases, and the foil begins to cool. The maximum  $\alpha$  is attained, as would be expected, for  $\varepsilon$  of 0.7–0.8 when the foil is in contact with the hydrodynamic wake behind the bubble, i.e., where the gas and particle fluxes are most vigorous. If the heat-transfer surface only rarely comes in contact with close-packed clumps of largely immobile particles ( $\varepsilon \approx \varepsilon_0$ ), the instantaneous heat-transfer coefficient in such instances is less than the mean value.

The recordings for the porosity near the wall and the films show that local areas on the surface of a vertical plate are sometimes in contact with bubbles free from particles (the more frequently the larger the particle diameter and the larger the fluidization number). However, when the mean speed of rise of the bubbles is 0.3–0.4 m/sec, the time of contact between the surface and the natural boundary of a rounded bubble does not exceed 0.005–0.010 sec. No boundary layer can be set up in this time, and the effective boundary layer is as thin as during the contact with the bed of particles, while the heat-transfer coefficient remains as high as before [24].

Our results agree well with those of [8, 9, 11, 23] but deviate appreciably from those of [7, 14]; Table 2 gives observed values for  $\alpha_{\min}/\alpha_{\max}$  as recorded under identical conditions and with identical values for  $(c_p \delta)_{ef}$ . It is clear that the results of [7, 14] are not reproduced under comparable conditions. There is an error in [7, 14] in the test made on the bunching theory, which was overlooked by those workers, since on substitution into the equation

$$\bar{\alpha} = \bar{\alpha}_0 f_0 + (1 - f_0) \frac{1}{R_k + 0.47R_k} \quad (4)$$

values of  $\alpha_0$  too low by an order of magnitude, the error in determining  $\bar{\alpha}$  was balanced out by the underestimation of  $R_k$ , which created the impression that the bunching model was reliable.

In conclusion, we may note the dual mode of external heat transfer in a fluidized bed; as in the case of a surface flushed by an eddy in a homogeneous liquid (rear zone of a cylinder, sphere, or other part of a surface behind an obstacle), the heat transfer in a fluidized bed is essentially nonstationary for a surface whose size is comparable with the particle diameter, and in that case the approach of [16] is applicable. On the other hand, the heat transfer between a fluidized system and an extended surface should be considered as quasistationary when the mean characteristics of the temperature and velocity distributions are independent of time. Here we may note that the high heat-transfer rate in pulsating dispersed media cannot be explained in terms of an essentially nonstationary convective process [17] or from the viewpoint of the bunching theory.

We thus propose a new hypothesis for dispersed systems on the basis of our observations that fast fluctuations in a fluidized system have little effect on the local instantaneous values for the heat-flux density and heat-transfer coefficient: the boundary layers deformed by the particles or particle clumps are essentially nonstationary, dynamic, and fluctuating, especially during contact of the surface with the outer part of a gas bubble, or the hydrodynamic wake behind one, and such boundary layers remain extremely thin, which results in uniform and high rates of heat transfer throughout the cycle.

## NOTATION

$I$ , current, A;  $t_b$ , temperature of bed core, deg;  $\vartheta$ ,  $\bar{\vartheta}$ , instantaneous and mean excess surface temperatures, deg;  $W$ , fluidization number;  $Bi$ , Biot number;  $c$ ,  $\rho$ , specific heat (J/kg·deg) and density (kg/m<sup>3</sup>) of plate;  $\tau$ , time, sec;  $V$ ,  $F$ , volume (m<sup>3</sup>) and side surface (m<sup>2</sup>) of platinum foil;  $Q_{sou}$ , power of internal heat sources, W;  $Q_{loss}$ , heat lost through probe base, W;  $\alpha$ ,  $\bar{\alpha}$ , instantaneous and mean heat-transfer coefficients, W/m<sup>2</sup>·deg;  $(c\rho\delta)_{ef}$ , effective integrating parameter of foil plus substrate, J/m<sup>2</sup>·deg;  $d$ , particle diameter, mm;  $\varepsilon$ ,  $\bar{\varepsilon}$ , instantaneous and mean bed porosities;  $q_{fl}$ ,  $\bar{q}_{fl}$ , instantaneous and mean heat flux densities, W/m<sup>2</sup>;  $\bar{\alpha}_0$ , mean coefficient of heat transfer to gas;  $R_\lambda$ , thermal resistance of heated particle bunch;  $f_0$ , relative contact time.

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